The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

- An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel.
- A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties

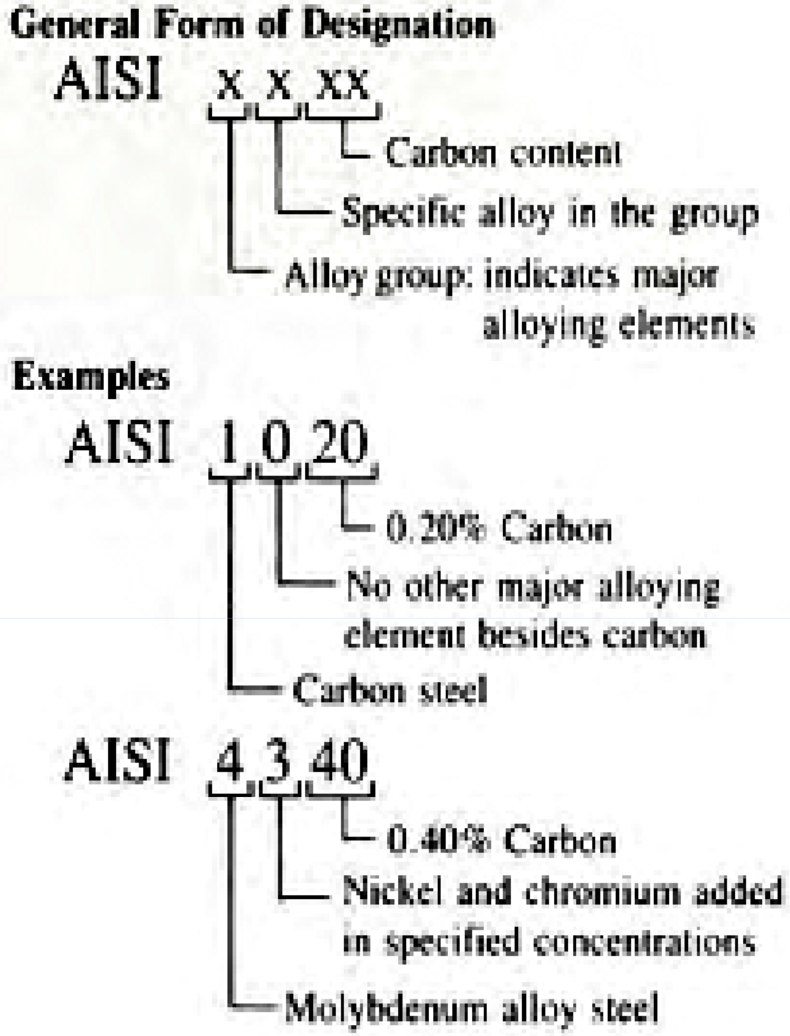
When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Select the material from which the shaft will be made, and specify its condition:

cold-drawn, heat treated, and soon. Refer to Table 2- 4 in Chapter 2 for suggestions for steel materials for shafts. Plain carbon or alloy steels with medium carbon content are typical, such as AISI 1040, 4140, 4340, 4640, 5150, 6150, an d 8650. Good ductility with percent elongation above about 12% is recommended.

Determine the ultimate strength, yield strength, and percent elongation of the selected material.

FIGURE 2–11 Steel designation system



elements present in the various alloy steels are sulfur, phosphorus, silicon, nickel, chromium, molybdenum, and vanadium.

TABLE 2–3 Alloy groups in the AISI numbering system

10xx	Plain carbon steel: No significant alloying element except carbon and manganese; less than 1.0% manganese. Also called *nonresulfurized*.
11xx	Free-cutting steel: Resulfurized. Sulfur content (typically 0.10%) improves machinability.
12xx	Free-cutting steel: Resulfurized and rephosphorized. Presence of increased sulfur and phosphorus improves machinability and surface finish.
12Lxx	Free-cutting steel: Lead added to 12xx steel further improves machinability.
13xx	Manganese steel: Nonresulfurized. Presence of approximately 1.75% manganese increases hardenability.
15xx	Carbon steel: Nonresulfurized; greater than 1.0% manganese.
23xx	Nickel steel: Nominally 3.5% nickel.
25xx	Nickel steel: Nominally 5.0% nickel.
31xx	Nickel-chromium steel: Nominally 1.25% Ni; 0.65% Cr.
33xx	Nickel-chromium steel: Nominally 3.5% Ni; 1.5% Cr.
40xx	Molybdenum steel: 0.25% Mo.
41xx	Chromium-molybdenum steel: 0.95% Cr; 0.2% Mo.
43xx	Nickel-chromium-molybdenum steel: 1.8% Ni; 0.5% or 0.8% Cr; 0.25% Mo.
44xx	Molybdenum steel: 0.5% Mo.
46xx	Nickel-molybdenum steel: 1.8% Ni; 0.25% Mo.
48xx	Nickel-molybdenum steel: 3.5% Ni; 0.25% Mo.
5xxx	Chromium steel: 0.4% Cr.
51xx	Chromium steel: Nominally 0.8% Cr.
51100	Chromium steel: Nominally 1.0% Cr; bearing steel, 1.0% C.
52100	Chromium steel: Nominally 1.45% Cr; bearing steel, 1.0% C.
61xx	Chromium-vanadium steel: 0.50%–1.10% Cr; 0.15% V.
86xx	Nickel-chromium-molybdenum steel: 0.55% Ni; 0.5% Cr; 0.20% Mo.
87xx	Nickel-chromium-molybdenum steel: 0.55% Ni; 0.5% Cr; 0.25% Mo.
92xx	Silicon steel: 2.0% silicon.
93xx	Nickel-chromium-molybdenum steel: 3.25% Ni; 1.2% Cr; 0.12% Mo.

TABLE 2–4 Uses of some steels

UNS number	AISI number	Applications
G10150	1015	Formed sheet-metal parts; machined parts (may be carburized)
G10300	1030	General-purpose, bar-shaped parts, levers, links, keys
G10400	1040	Shafts, gears
G10800	1080	Springs; agricultural equipment parts subjected to abrasion (rake teeth, disks, plowshares, mower teeth)
G11120	1112	Screw machine parts
G12144	12L14	Parts requiring good machinability
G41400	4140	Gears, shafts, forgings
G43400	4340	Gears, shafts, parts requiring good through-hardening
G46400	4640	Gears, shafts, cams
G51500	5150	Heavy-duty shafts, springs, gears
G51601	51B60	Shafts, springs, gears with improved hardenability
G52986	E52100	Bearing races, balls, rollers (bearing steel)
G61500	6150	Gears, forgings, shafts, springs
G86500	8650	Gears, shafts
G92600	9260	Springs

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view :

1. ***Transmission shafts***. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to <u>bending in addition to twisting</u>.
2. ***Machine shafts***. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts :

1. *Shear stresses* due to the transmission of torque (i.e. due to torsional load).
2. *Bending stresses* (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a) 112 MPa for shafts without allowance for keyways.

(b) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σt) may be taken as 60 per cent of the elastic limit in tension

(σel), but not more than 36 per cent of the ultimate tensile strength (σu).

In other words, the permissible tensile stress,

$\sigma t = 0.6\ \sigma el$ or $0.36\ \sigma u$, whichever is less.

The maximum permissible shear stress may be taken as

(a) 56 MPa for shafts without allowance for key ways.

(b) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σel) but not more than 18 per cent of the ultimate tensile strength (σu). In other words, the permissible shear stress, $\tau = 0.3\ \sigma\ el$ or $0.18\ \sigma\ u$, whichever is less.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$T/J = r/\tau$$

T= Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ= Torsional shear stress, and

r = Distance from neutral axis to the outer most fiber =d/ 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3$$

From this equation, we may determine the diameter of round solid shaft (*d*).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation *(i)*, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...(iii)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation *(iii)* may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad \text{...(iv)}$$

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : N = 200 r.p.m. ; P = 20 kW = 20×10^3 W; τ = 42 MPa = 42 N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or $d = 48.7$ say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

$$\therefore \quad d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm}$$ **Ans.**

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.84 = 108\ 032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

$\therefore$ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\ 060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

14.10 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \text{...}(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4\,(1 - k^4) \qquad \text{...(where } k = d_i / d_o)$$

and $y = d_o / 2$

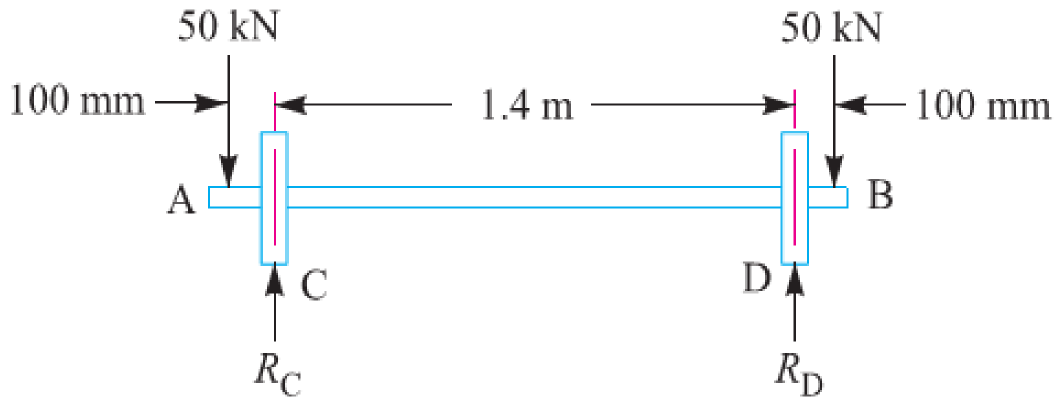
Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4\,(1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Example 14.4. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50\text{ kN} = 50 \times 10^3\text{ N}$; $L = 100\text{ mm}$; $x = 1.4\text{ m}$; $\sigma_b = 100\text{ MPa} = 100\text{ N/mm}^2$



The maximum B.M. may be obtained as follows :

$$R_C = R_D = 50\text{ kN} = 50 \times 10^3\text{ N}$$

B.M. at *A*, $M_A = 0$

D

B.M. at *C*, $M_C = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

B.M. at *D*, $M_D = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6$ N-mm

B.M. at *B*, $M_B = 0$

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$\therefore$ $d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$ say 80 mm **Ans.**

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ= Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

or

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad \text{...}(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad \text{...}(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \quad \text{...(iv)}$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \quad \text{...(v)}$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and $$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

From the equations **(*iii*)** or **(*iv*)**, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW $= 20 \times 10^3$ W; $\tau = 42$ MPa $= 42$ N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\ 733 \quad \text{or} \quad d = 48.7 \text{ say } 50 \text{ mm}$$ **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

∴ $$d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or $$d = 159.4 \text{ say } 160 \text{ mm}$$ **Ans.**

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : M = 3000 N-m = 3×10^6 N-mm ; T = 10 000 N-m = 10×10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \quad \text{or} \quad d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\ d^3$$

$\therefore$ $d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

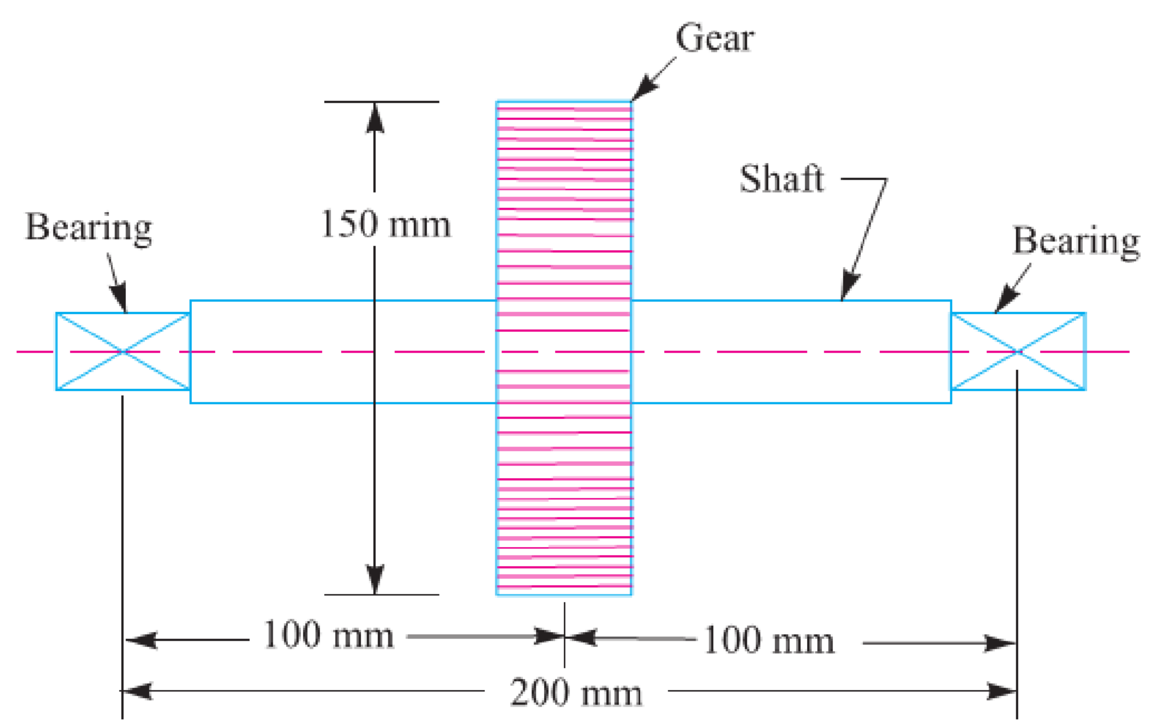
Taking the larger of the two values, we have

$d = 86$ say 90 mm **Ans.**

Example 14.6. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.



We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20°} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

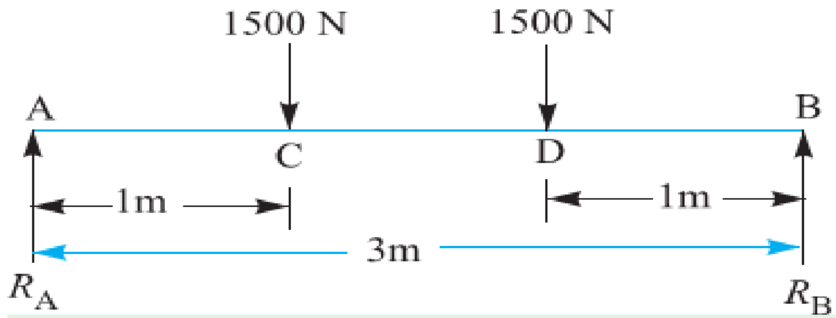


Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

∴ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

∴ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm **Ans.**

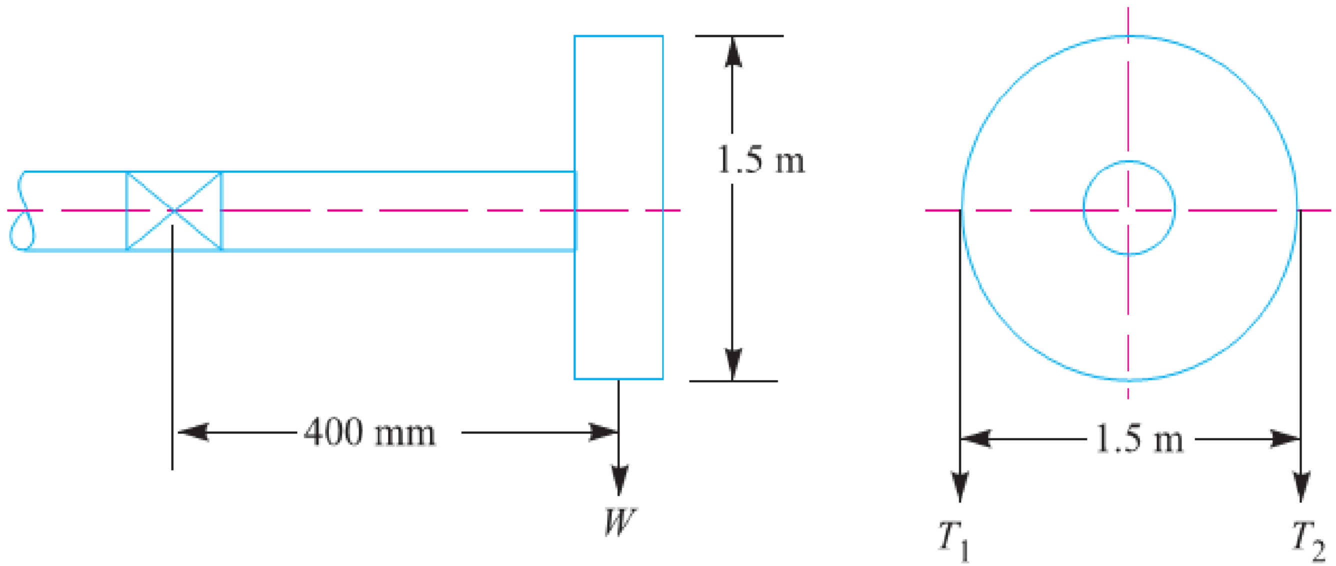
Example 14.8. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

Solution . Given : $D = 1.5$ m or $R = 0.75$ m; $T_1 = 5.4$ kN $= 5400$ N ; $T_2 = 1.8$ kN $= 1800$ N ; $L = 400$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2

A line shaft with a pulley is shown in Fig 14.4.

We know that torque transmitted by the shaft,

$$T = (T_1 - T_2)\, R = (5400 - 1800)\, 0.75 = 2700 \text{ N-m}$$
$$= 2700 \times 10^3 \text{ N-mm}$$



Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

$\therefore$ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$
$$= 3950 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 3950 \times 10^3 / 8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

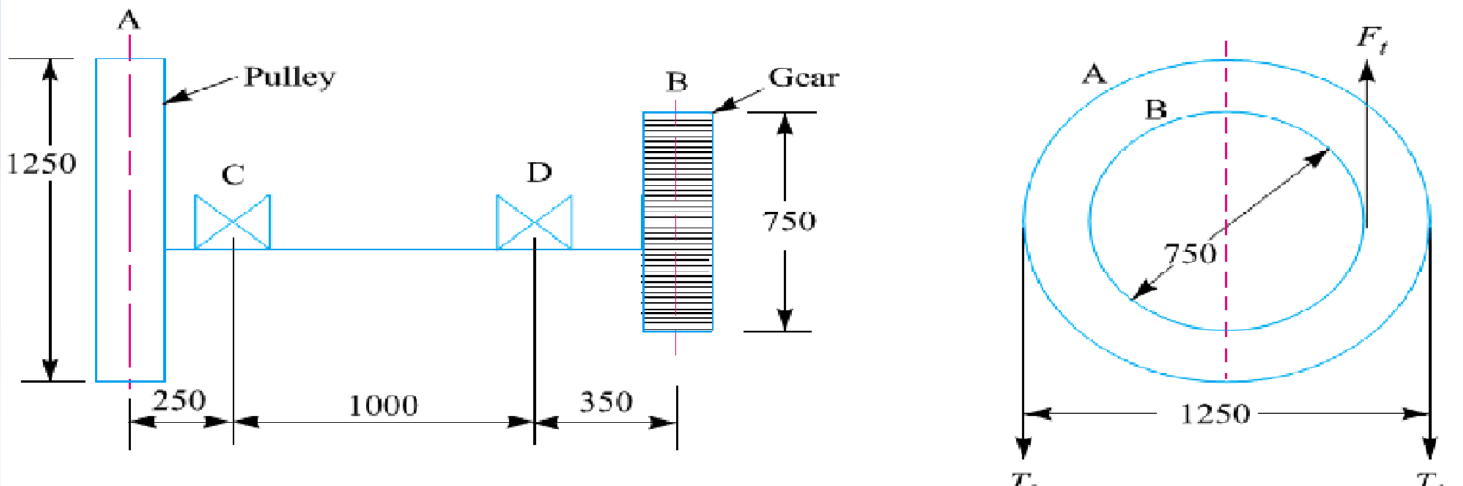
Example 14.14. *Fig. 14.10 shows a shaft carrying a pulley A and a gear B and supported in two bearings C and D. The shaft transmits 20 kW at 150 r.p.m. The tangential force F_t on the gear B acts vertically upwards as shown.*

The pulley delivers the power through a belt to another pulley of equal diameter vertically below the pulley A. The ratio of tensions T_1 / T_2 is equal to 2.5. The gear and the pulley weigh 900 N and 2700 N respectively. The permissible shear stress for the material of the shaft may be taken as 63 MPa. Assuming the weight of the shaft to be negligible in comparison with the other loads, determine its diameter. Take shock and fatigue factors for bending and torsion as 2 and 1.5 respectively.

Solution. Given : P = 20 kW = 20×10^3 W ; N = 150 r.p.m. ; $T_1/T_2 = 2.5$; W_B = 900 N ; W_A = 2700 N ; τ = 63 MPa = 63 N/mm^2 ; $K_m = 2$; $K_t = 1.5$; D_B = 750 mm or R_B = 375 mm ; D_A = 1250 mm or R_A = 625 mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 150} = 1273 \text{ N-m} = 1273 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt on pulley A.

Since the torque on the pulley is same as that of shaft (*i.e.* 1273×10^3 N-mm), therefore

$$(T_1 - T_2) R_A = 1273 \times 10^3 \quad \text{or} \quad T_1 - T_2 = 1273 \times 10^3 / 625 = 2037 \text{ N} \qquad \text{...(i)}$$

Since $T_1 / T_2 = 2.5$ or $T_1 = 2.5\, T_2$, therefore

$$2.5\, T_2 - T_2 = 2037 \quad \text{or} \quad T_2 = 2037/1.5 = 1358 \text{ N} \qquad \text{...[From equation (i)]}$$

and $T_1 = 2.5 \times 1358 = 3395$ N

$\therefore$ Total vertical load acting downward on the shaft at A

$$= T_1 + T_2 + W_A = 3395 + 1358 + 2700 = 7453 \text{ N}$$

Assuming that the torque on the gear B is same as that of the shaft, therefore the tangential force acting vertically upward on the gear B,

$$F_t = \frac{T}{R_B} = \frac{1273 \times 10^3}{375} = 3395 \text{ N}$$

Since the weight of gear B (W_B = 900 N) acts vertically downward, therefore the total vertical load acting upward on the shaft at B

$$= F_t - W_B = 3395 - 900 = 2495 \text{ N}$$

Now let us find the reactions at the bearings C and D. Let R_C and R_D be the reactions at C and D respectively. A little consideration will show that the reaction R_C will act upward while the reaction R_D act downward as shown in Fig. 14.11.

For the equilibrium of the shaft,

$$R_D + 7453 = R_C + 2495 = 10\,200 + 2495 = 12\,695$$

$$\therefore \quad R_D = 12\,695 - 7453 = 5242 \text{ N}$$

We Know that B.M. at A and B

$$= 0$$

B.M. at C $\quad = 7453 \times 250 = 1863 \times 10^3$ N-mm

B.M. at D $\quad = 2495 \times 350 = 873 \times 10^3$ N-mm

We see that the bending moment is maximum at C.

$$\therefore \quad \text{Maximum B.M.} = M = M_C = 1863 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1863 \times 10^3)^2 + (1.5 \times 1273 \times 10^3)^2}$$

$$= 4187 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$4187 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 63 \times d^3 = 12.37\, d^3.$$

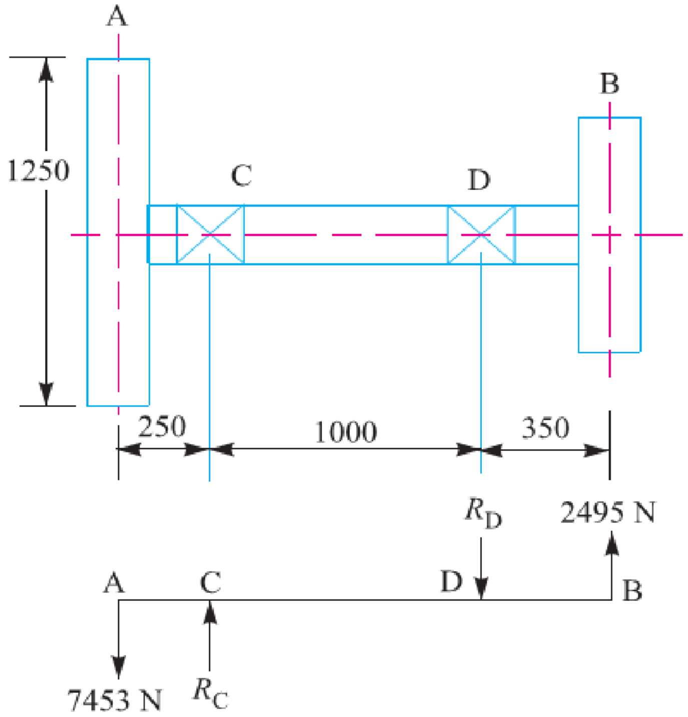
$$\therefore \quad d^3 = 4187 \times 10^3 / 12.37 = 338 \times 10^3$$

$d = 69.6$ say 70 mm **Ans.**

nts about D, we get

$$R_C \times 1000 = 7453 \times 1250 + 2495 \times 350 = 10.2 \times 10^6$$

$$R_C = 10.2 \times 10^6 / 1000 = 10\,200 \text{ N}$$

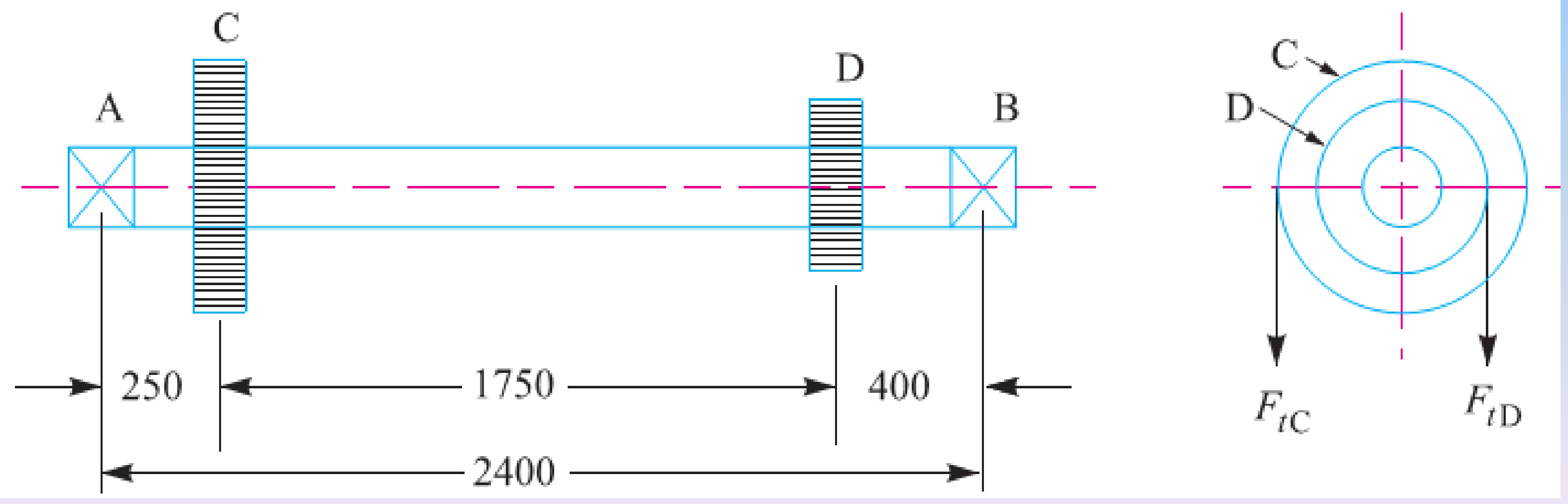


Example 14.15. *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.*

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Solution. Given : $AC = 250$ mm ; $BD = 400$ mm ; $D_C = 600$ mm or $R_C = 300$ mm ; $D_D = 200$ mm or $R_D = 100$ mm ; $AB = 2400$ mm ; $P = 20$ kW $= 20 \times 10^3$ W ; $N = 120$ r.p.m ; $\sigma_t = 100$ MPa $= 100$ N/mm^2 ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $W_C = 950$ N ; $W_D = 350$ N ; $K_m = 1.5$; $K_t = 1.2$

The shaft supported in bearings and carrying gears is shown in Fig. 14.12.



We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears C and D is same as that of the shaft, therefore the tangential force acting at gear C,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

and total load acting downwards on the shaft at C

$$= F_{tC} + W_C = 5300 + 950 = 6250 \text{ N}$$

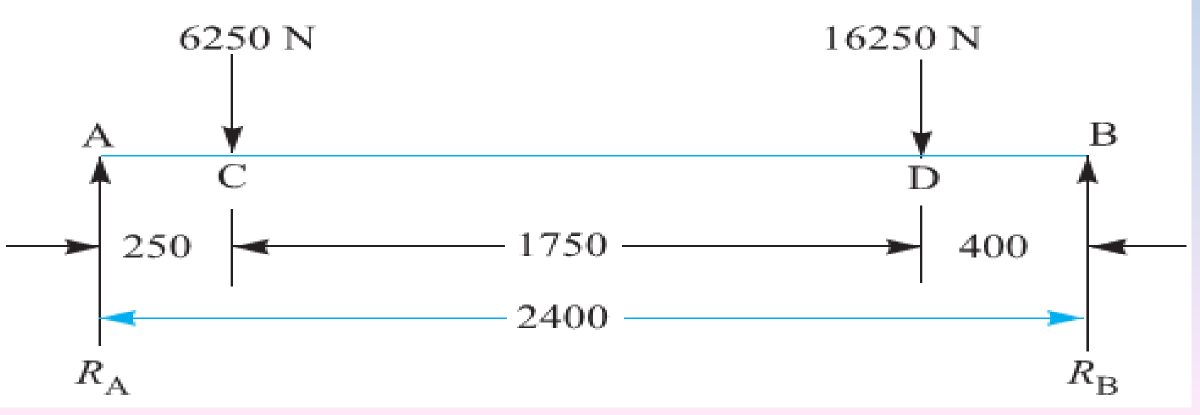
Similarly tangential force acting at gear D,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at D

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250 \text{ N}$$

Now assuming the shaft as a simply supported beam as shown in Fig. 14.13, the maximum bending moment may be obtained as discussed below :



Let R_A and R_B = Reactions at A and B respectively.

$\therefore$ $R_A + R_B$ = Total load acting downwards at C and D

$$= 6250 + 16\,250 = 22\,500 \text{ N}$$

Now taking moments about A,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

$$\therefore \quad R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190 \text{ N}$$

and $\quad R_A = 22\,500 - 14\,190 = 8310 \text{ N}$

A little consideration will show that the maximum bending moment will be either at C or D.

We know that bending moment at C,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 \text{ N-mm}$$

Bending moment at D,

$$^*M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

$\therefore$ Maximum bending moment transmitted by the shaft,

$$M = M_D = 5676 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$$= 8725 \times 10^3 \text{ N-mm}$$

The bending moment at D may also be calculated as follows :

$M_D = R_A \times 2000 - (\text{Total load at } C)\,1750$

We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore \qquad d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \ \text{ or } \ d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\,(K_m \times M + T_e)$$

$$= \tfrac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right] = 8620 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\ d^3 \qquad \text{...(Taking } \sigma_b = \sigma_t)$$

$$\therefore \qquad d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3 \ \text{ or } \ d = 95.7 \text{ mm}$$

Taking the larger of the two values, we have

$d = 95.7$ say 100 mm **Ans.**

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σb). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\,d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... } (\because\ k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi\,d^3} + \frac{4\,F}{\pi\,d^2} = \frac{32}{\pi\,d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32\,M_1}{\pi\,d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32\,M_1}{\pi (d_o)^3\,(1 - k^4)}$$

$$\text{...}\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1 + k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 (1 - k^2)} \qquad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044\,(L/K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

$$\text{**Column factor,} \quad \alpha = \frac{\sigma_y (L/K)^2}{C \pi^2 E}$$

where
L = Length of shaft between the bearings,
K = Least radius of gyration,
σ_y = Compressive yield point stress of shaft material, and
C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C = 1, for hinged ends,
= 2.25, for fixed ends,
= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha\, F\, d_o\,(1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F\, d_o\,(1+k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F\, d_o\,(1+k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1-k^4)$$

It may be noted that for a solid shaft, $k = 0$ and $d_0 = d$. When the shaft carries no axial load, then $F = 0$ and when the shaft carries axial tensile load, then $\alpha = 1$.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. Torsional rigidity. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per meter length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per meter length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G\,.\,\theta}{L} \quad \text{or} \quad \theta = \frac{T\,.\,L}{J\,.\,G}$$

θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity.* It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then

the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW $= 4000$ W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m $= 1000$ mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

$\therefore$ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\ (35)^3 = 8420\ \tau$$

$\therefore$ $\tau = 47\ 740 / 8420 = 5.67$ N/mm² $= 5.67$ MPa **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \qquad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \qquad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation ***(i)*** by equation ***(ii)***, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \qquad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75$$ **Ans.**

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \, (d_o)^3 \; (1 - k^4) \qquad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3 \; (1 - k^4)}{d^3} = \frac{(d_o)^3 \; (1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because \; d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \; \textbf{Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right] \qquad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because \; d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \; \textbf{Ans.}$$